

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

M.A./M.SC. THIRD SEMESTER EXAMINATION, DECEMBER 2012

SECOND YEAR

APPLIED CHEMISTRY

Date : 10/12/2012

Time : 11 am – 1 pm

Paper : XI (Engineering Mathematics)

Full Marks : 50

(Use separate answer-script for each group)

Group – A

(Answer any six of the following)

[6x5]

1. a) The eigen values of a square matrix A of order 3 over $\mathbb{R}$ are in Arithmetic progression. If the trace of matrix A be 9 and $\det A = 15$, determine the eigen values. [2]

- b) Examine whether the following system of linear equations is consistent or not: [3]

$$x - 5y + 3z = -1$$

$$2x - y - z = 5$$

$$5x - 7y + z = 2$$

2. a) Let $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$.

Given that 4 is an eigen value of A , find the corresponding eigen vector. [2]

- b) Examine whether the matrix $A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & -7 & 2 \\ 0 & 0 & 4 \end{pmatrix}$ is diagonalisable. [3]

3. a) Examine whether the series $\sum_n \frac{1}{(1+n)\sqrt{n}}$ is convergent. [2]

- b) Pick out the correct answer with reason:

$\lim_{x \rightarrow 0} \left(\frac{1}{\tan x} - \frac{1}{x} \right)$ is (i) $-\infty$ (ii) ∞ (iii) 0 (iv) none of these. [3]

4. a) With justification, choose the correct answer:

If f be continuous over $[0, 20]$, then $I = \int_0^{20} \frac{f(x)}{f(x) + f(20-x)} dx$ is equal to (A) 15 (B) 10 (C) 20 (D) 30. [2]

- b) If $u = \sin^{-1} \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$. [3]

5. Answer either (a) and (b) or (c):

- a) Show that $f(x, y) = xy(a - x - y)$ has an extreme value at $\left(\frac{a}{3}, \frac{a}{3} \right)$ where 'a' is non-zero real number. [3]

- b) Show that $\log(1+x) > \frac{x}{1+x}$ for all $x > 0$. [2]

- c) Evaluate $\iint x^{\frac{1}{2}} y^{\frac{1}{2}} (1-x-y)^{\frac{1}{2}} dx dy$ over the interior of the triangle formed by the lines $x=0, y=0$ & $x+y=1$. [5]

6. Show that $\frac{1}{y^2}$ is an Integrating factor of the ODE $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ and hence solve it. [2+3]

7. a) Find the particular Integral: $\frac{d^2 y}{dx^2} - 9 \frac{dy}{dx} + 20y = e^{2x}$. [2]

b) Solve: $x^2 \frac{d^2 y}{dx^2} + 5x \frac{dy}{dx} + 4y = 0$. [3]

8. Answer either (a) or (b): [5]

a) For the function $f(z)$ defined by

$$f(z) = \begin{cases} \frac{(\bar{z})^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

Show that Cauchy-Riemann equations are satisfied at $(0, 0)$ but the function is not differentiable at $(0, 0)$.

b) Correct or justify: $f(z) = x^2 y^2 + i \cdot 2x^2 y^2$ is nowhere analytic.

9. Using Residue theorem, evaluate $\int_C \frac{2+3\sin \pi z}{z(z-1)^2} dz$ where C is a square having vertices at $3+3i, 3-3i, -3+3i, -3-3i$. [5]

10. Using Cauchy's integral formula, evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 2$. [5]

Group – B

(Answer **any two** of the following)

11. Answer **any five**: 5x2

i) Show that two proper events can not be simultaneously mutually exclusive and independent.

ii) Show that the probability of obtaining no head in an infinite sequence of independent tosses of a coin is zero.

iii) If A and B be two mutually exclusive events and $P(A+B) \neq 0$, prove that

$$P(A|(A+B)) = \frac{P(A)}{P(A)+P(B)}.$$

iv) Let A and B be two events such that $P(A) = \frac{3}{4}$, $P(B) = \frac{5}{8}$. Show that $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$.

v) Find the value of the constant k so that the function

$$f_x(x) = \begin{cases} kx(2-x), & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function.

vi) Show that the distribution function F_x of a random variable X is non-decreasing.

vii) For any three events A, B, C show that $P(A+B+C) \leq P(A)+P(B)+P(C)$.

12. Using the following table:

x :	0.30	0.32	0.34	0.36	0.38	0.40
$f(x)$:	1.760	1.770	1.780	1.791	1.802	1.814

Compute (i) $f(0.31)$ and (ii) $f(0.39)$, correct to 3 decimal places. [5+5]

13. i) Prove that $r^n \vec{r}$ is irrotational for every n . Find n when it is solenoidal. [3+3]

ii) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ [4]

OR

If $\vec{F} = (2x^2 + y^2)\hat{i} + (3y - 4x)\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ around the triangle ABC whose vertices are

$A(0, 0)$, $B(2, 0)$ and $C(2, 1)$. [4]

